What does it mean to modify or test Newton's second law?

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Physicists in the 19th century concluded that Newton's second law is not a real law of nature but a mere definition, because force cannot be measured directly but only through the acceleration it produces on a given mass. Although the debate seemed settled, it has been proposed in recent years that Newton's second law could be modified, as if it were a real description of nature, to explain some intriguing aspects of galactic dynamics. Experiments have been performed claiming to test the validity of the second law. The aim of this article is to elucidate, without using the concept of force, what it means to modify or experimentally test Newtonian dynamics.

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I. INTRODUCTION

In the century following Newton's statement of his celebrated laws of motion, physicists and philosophers debated whether force was a primary concept or a mere aid in calculations. After long discussions, physicists at the end of the 19th century reached the consensus that the relation $F=ma$ was just a definition, because the only quantity that can be measured unambiguously is the acceleration. The force can be measured only through the acceleration it produces on a body whose mass is assumed to be known. Alternatively, if we assume F to be known, then the mass is determined as the ratio F/a. Kirchhoff argued that the second law means that the basic equations of mechanics should involve second derivatives of the position because this form is the simplest possibility, given that the motion of a particle depends on its initial position and velocity, but not on its initial acceleration.[1] Mach denied that the concept of force had any meaning and argued that the only consistent way to define mass is as the inverse of the acceleration that one body causes another to have.[2] Poincaré also thought that Newton's second law is a definition of force. As for mass, he examined all attempts to give it a clear and precise definition, but to his regret reached the conclusion that "masses are coefficients which it is found convenient to introduce into calculations."[3] Most physicists who have thought about the subject share the same opinion, as documented by Jammer.[4]

Jammer concluded that force is a useful concept only as a "methodological intermediate" in describing the motion induced by a physical system, and expresses the fact that the product ma for all test particles is a function of the configuration of a particular system. More recently, Wilczek[5] argued that this law is "presented as an algorithm describing nature …, but it is more like a language in which we can easily express important facts about the world." The concept of force has disappeared from relativity and quantum mechanics, and it is well known that mass is not strictly constant.

Notwithstanding this historical and conceptual background, many elementary physics textbooks treat $F=ma$ as a fundamental law. This treatment is not surprising because its conceptual usefulness is undeniable. However, a physics teacher may be questioned by well informed students who have heard of contemporary attempts to modify this law and to test it experimentally. In particular, the formulation of modified Newtonian dynamics (MOND)[6,7] has attracted much attention in recent years as a plausible alternative to "dark matter." Given the interest in this dynamics, there have been at least two experiments claiming to test for departures from Newton's law in a terrestrial laboratory: Abramovici and Vager[8] used test masses driven by small electric currents, and more recently Gundlach *et al.*[9] used a torsion balance. The results were negative in both experiments. A much more sophisticated experiment has been proposed by Ignatiev[10] and might be realized in the future.

The aim of this paper is to clarify what it means to modify and test Newton's second law experimentally. In Sec. II the MOND model is analyzed from a general point of view that does not employ the controversial concept of force. In Sec. III the theoretical basis of the experiment reported in Ref. 9 is analyzed to elucidate whether it is a confirmation of Newton's law, or by a circular argument, just a test of Hooke's law.

II. MODIFYING NEWTONIAN DYNAMICS

The original purpose of MOND was to give a simple explanation of the observed rotation curves of galaxies.[6,7] We briefly summarize the problem and the proposed solutions. It is known that the velocity of rotation of stars in galaxies does not decrease with distance according to Kepler's third law as if the mass of the galaxy were concentrated at its center. Instead, this velocity remains more or less constant up to the very galactic edge. This relation between velocity and orbital radius is usually taken as evidence that very massive halos made up of a mysterious dark matter surround the galaxies. However, Milgrom[6,7] showed that the same observations could also be explained by a slight and simple modification of Newton's second law. The basic idea is that the acceleration of outer stars in a galaxy is of the order of $10^{-10}\ \mathrm{m/s^2}$, but there is no experimental evidence that Newton's laws apply for such small accelerations. Kepler's law, $v \propto r^{-1/2}$, relating the velocity v to the distance r to the attracting center, follows from Newton's second law and the gravitational force law,

$$F = ma = m\frac{v^2}{r} = \frac{GMm}{r^2}. \tag{1}$$

As noted by Milgrom,[6,7] if the force in the left equality were proportional to a^2, then v would be independent of r in accordance with astronomical observations, without any need

to modify the gravitational force or postulate dark matter. A way to express this idea in a more sophisticated manner is to write Newton's second law as[6,7]

$$F = mf(a/a_0)a, \tag{2}$$

where a_0 is a very small acceleration and $f(a/a_0)$ is an unspecified function with the property that $f(a/a_0)=1$ if $a \gg a_0$ and $f(a/a_0)=a/a_0$ if $a \ll a_0$. In practice, a_0 should be of the order of magnitude of 10^{-10} m/s^2, which is of the order of the velocity of light multiplied by the Hubble constant.

However, gravitational force, mass, and acceleration are not independently measurable physical quantities. The mass of a gravitating body cannot be deduced from its motion because all bodies fall with the same acceleration due to the equivalence between gravitational and inertial mass. This equivalence is very clear in general relativity, where the acceleration of test particles is given by geodesic equations that do not involve their mass. Thus, if we forget about mass and avoid using the concept of force, the MOND formalism turns out to be just a convenient way to state that the acceleration produced by gravity is $a \propto r^{-1}$ for $a \approx a_0$. This modified gravitational law is not what general relativity predicts, but something of the sort is possible in more general theories.[11]

III. TESTING NEWTONIAN DYNAMICS

Can Newton's second law be tested using a pendulum or some other oscillating device? In the experiment reported in Ref. 9 a very sensitive torsion balance was used to evaluate the relation between the supposedly known forces and the observed accelerations. According to Newton's law, such a relation should be linear with the constant of proportionality given by the mass of the oscillating body. Gundlach *et al.*[9] assumed that any departure from a linear relation for very small amplitudes could be interpreted as evidence in favor of MOND. No discrepancy was observed for accelerations as small as 10^{-13} m/s^2.

However, only length and time intervals can be measured directly, while the force must be inferred from an expression that defines it in terms of mass, which must be deduced independently. In particular, the motion of a harmonic oscillator according to Newton's second law is given by $F=m\ddot{x}=-kx$, where k is Hooke's constant, and the maximum force $F_{\max}$ and the acceleration $a_{\max}$ are related to the amplitude A and frequency ω by $F_{\max}=kA$ and $a_{\max}=A\omega^2$. The ratio $F_{\max}/a_{\max}$ is just the mass m of the oscillator and must be constant if Newton's second law applies. Now a question arises: Is not the whole argument tautological because only amplitudes and periods can be measured directly? The mass of the oscillator can be measured with a balance, but there is no way to deduce it from its motion; as for Hooke's constant, it is only a convenient way to express the product $m\omega^2$ in Newtonian dynamics, where m is already given.

In the experiment of Ref. 9 the amplitude A and the period of oscillation ω of a torsion balance were measured. If the mass (or moment of inertia) is known independently, then Hooke's constant k (more precisely, the fiber torsional constant) can be deduced according to Newton's second law. In Ref. 9 a definite value of k was assumed for the torsion, and the maximum force and acceleration were deduced from the relations $F_{\max}=kA$ and $a_{\max}=A\omega^2$, respectively. A linear relation between $F_{\max}$ and $a_{\max}$ was found experimentally, meaning that there is a linear relation between acceleration and displacement. If the ratio $F_{\max}/a_{\max}$ had not turned out to be constant, a plausible conclusion would be that Hooke's law does not apply exactly, in contradiction to the explicit assumption of Ref. 9.

Hooke's law is only empirical and states the fact that close to the equilibrium configuration of a system, the response is linear. Accordingly, we can forget about forces and masses and state Hooke's law to be the claim that two measurable quantities, acceleration and displacement, are proportional to each other. But how should a MOND effect be interpreted without using the concepts of force or mass? A textbook result of classical mechanics is that the frequency of oscillations is constant and independent of the amplitude only if the acceleration depends linearly on the position, $\ddot{x} \propto x$ (we can still define a force as $m\ddot{x}$, but this definition is not necessary). If MOND [Eq. (2)] and Hooke's law are correct, we would expect an acceleration proportional to $x^{1/2}$ for an amplitude of oscillation of the order $x_0=a_0/\omega^2$, resulting in an amplitude dependence of the oscillation period.

Reference 9 reported a numerical calculation of the motion of a torsion balance with a nonlinear relation between angle and acceleration, but assuming that Hooke's constant (the torsional constant) remains constant for all values of the acceleration. As expected, they obtained a time varying period of oscillation different from the simple linear case. For the experiment they used a pendulum with a natural period of 795 s. Accordingly, the distance x_0 over which MOND effects might be expected would be $x_0 \sim 2\times 10^{-6}$ m for $a_0 \sim 10^{-10}$ m/s^2. Any alteration of the frequency of oscillation would imply either an apparent deviation from Newtonian dynamics if we assume Hooke's law to be valid, or a nonlinear response for very small accelerations if MOND is correct. The authors measured amplitudes as small as 4×10^{-13} m and found no such deviation: the motion of the torsion pendulum is still harmonic.

IV. CONCLUSIONS

The experiments performed so far have not revealed anything unusual for very small accelerations. In particular, Ref. 9 demonstrated that there is a linear relation between the acceleration of a torsion pendulum and its deviation from equilibrium even for very small oscillations. Given that the constant of proportionality between these two quantities is related to the mass, the experimental result is a confirmation of the "zeroth law" of classical mechanics: mass is conserved.[5] Had an experiment of this sort revealed some deviation from linearity, the result could also be interpreted as an indication that mass depends on acceleration or, more simply, that the torsional constant of the balance is not constant for too small an oscillation.

In summary, to explain the rotation of galaxies we can either assume that the Newtonian gravitational force law is valid but that Newtonian dynamics must be modified, or that the gravitational force law has to be modified. The first alternative, that of MOND, relies on the concept of force. To be consistent with experiments and preserve the harmonicity of small oscillations, it would also require an *ad hoc* modification of the Hooke's law. The second alternative can be formulated without the concept of force, as in general relativity, although Newton's second law can still be employed as a useful conceptual tool in the nonrelativistic regime.

The original purpose of MOND was to point out that gravity may still hold some surprises beyond general relativity. A

genuine experimental test would involve the gravitational force between masses, in addition to mechanical or electrical forces. Even if such a test were to validate the MOND prediction ($a \propto r^{-1}$ for $a \ll a_0$), that result would not require modifying Newton's second law; we could instead modify the gravitational force law (preferably on the basis of a new physical theory). Given that the concept of force is merely a "methodological intermediate"[4] for practical calculations, the choice of which law to use depends on simplicity. Not surprisingly, the experiment of Ref. 9 suggests that staying with Newton may be simpler.

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[1]R. Dugas, *A History of Mechanics* (Dover, New York, 1988), Chap. X.
[2]E. Mach, *The Science of Mechanics* (Open Court, La Salle, IL, 1989), Chap. II, Sec. 5.
[3]H. Poincaré, "Classical mechanics," in *Science and Hypothesis* (Walter Scott, London, 1905), Chap. 6.
[4]M. Jammer, *Concepts of Force* (Dover, New York, 1957).
[5]F. Wilczek, "Whence the force of $F=ma$? I: Culture shock," Phys. Today **57**(10), 11–12 (2004); "Whence the force of $F=ma$? II: Rationalizations," Phys. Today **57**(12), 10–11 (2004); "Whence the force of $F=ma$? III: Cultural diversity," Phys. Today **58**(7), 10–11 (2005).
[6]M. Milgrom, "A modification of the Newtonian dynamics as a possible alternative to the hidden mass hypothesis," Astrophys. J. **270**, 365–370 (1983).
[7]M. Milgrom, "A modification of the Newtonian dynamics: Implications for galaxies," Astrophys. J. **270**, 371–389 (1983).
[8]A. Abramovici and Z. Vager, "Test for Newton's second law at small accelerations," Phys. Rev. D **34**, 3240–3241 (1986).
[9]J. H. Gundlach, S. Schlamminger, C. D. Spitzer, K.-Y. Choi, B. A. Woodahl, and E. Fischbach, "Laboratory test for Newton's second law for small accelerations," Phys. Rev. Lett. **98**, 150801-1–4 (2007).
[10]A. Yu. Ignatiev, "Is violation of Newton's second law possible?," Phys. Rev. Lett. **98**, 101101-1–4 (2007).
[11]See, for example, J. D. Bekenstein, "Relativistic gravitation theory for the modified Newtonian dynamics paradigm," Phys. Rev. D **70**, 083509-1–28 (2004).

Inclined Plane Model. The inclined plane and the screw are usually presented as two different simple machines, but this paper model in the Millington-Barnard Collection at the University of Mississippi Museum shows clearly that they are really the same device. (Photograph and Notes by Thomas B. Greenslade, Jr., Kenyon College)